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## Sum of prime numbers

(suggested solutions)

Abstract: the author has improved the published articles.
about this theme. And I want to express my deep gratitude to the editors AMJS for interest and understanding of my work. In turn, the author strives to convey to reader important conclusions in his opinion, which in this article, namely that a transition from the ternary problem is possible to binary and further to solving the problem of infinity of numbers of twins.

Keywords: Proposal, solutions, to, current, problems

## Content:

1. Algebraic sum $4 x$ prime any even.
2. Sum of 2 prime numbers.
3. Number of representations by the sum of primes, both even and odd not one.
4.Sum of 2 or more prime numbers.
4. Gemini is endless.
5. Algebraic sum $4 x$ prime any even.

The sum of four primes - any even number does not require special proof if the ternary problem is proven:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+3=2 \mathrm{~K}+1+3=2(K+2) \tag{01}
\end{equation*}
$$

where $K \geq 3$, and taking into account 4 twos, then starting from 8 all numbers are even. Let us prove that the sum of 4 primes is any even number with the smaller equal to 8.

Let the sum of four primes be equal to some even $2 N$.

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}=2 \mathrm{~N} \tag{02}
\end{equation*}
$$

where is the integer $\quad N \geq 4 \quad$ ( $N$-not any)
Add 1 to both sides:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+1=2 \mathrm{~N}+1 \tag{03}
\end{equation*}
$$

According to the ternary problem, the right side (03) is the sum of three starting from $9=3+3+3$ :

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+1=p_{5}+p_{6}+p_{7} \tag{04}
\end{equation*}
$$

Adding 1 again:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+2=p_{5}+p_{6}+p_{7}+1 \tag{05}
\end{equation*}
$$

We replace the sum $\quad p_{5}+p_{6}+1=p_{8}+p_{9}+p_{10} \quad$ with the smallest $7=2+2+3$.
Now we have:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+2=p_{7}+p_{8}+p_{9}+p_{10} \tag{06}
\end{equation*}
$$

Which indicates that all even numbers are previous and subsequent and in ()

$$
\text { integer } N \geq 4 \text { - any! }
$$

By algebraic sum we mean that in four there are from 1 to $3 x$ primes can have a minus sign. And then the total algebraic sum starts with 2. This is obvious from:

$$
\begin{equation*}
p_{1}-p_{2}-p_{3}-p_{4}+2=p_{5}-p_{6}-p_{7}-p_{8} \tag{07}
\end{equation*}
$$

if we swap the prime numbers, we get the sum of 4 prime numbers equal to
any even number starting from 8.

## 2. Sum of 2 prime numbers

The sum of 2 prime numbers is any even number, starting from 4. According to above:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}=2 \mathrm{~N} \tag{08}
\end{equation*}
$$

where is the integer $\quad N \geq 4$
from which it follows that when $2 \mathrm{~N}=2\left(p_{1}+p_{2}\right)+2$ имеем:

$$
\begin{equation*}
p_{3}+p_{4}=p_{1}+p_{2}+2 \tag{09}
\end{equation*}
$$

which means the next even number is the sum of two primes the sum of two primes is the previous even number plus two.

Substituting instead of $p_{1,} p_{2}$ the values of $p_{3,} p_{4}$ we have an infinite series of all even 4, 6... numbers without exception!

Let us assume inequality (09). Then, according to the sum of four, any even:

$$
\begin{equation*}
p_{5}+p_{6}+p_{7}+p_{8} \neq p_{1}+p_{2}+p_{3}+p_{4} \tag{10}
\end{equation*}
$$

from which it follows:

$$
\begin{align*}
& p_{8}-p_{4}=2 \mathrm{~N}_{2}  \tag{11}\\
& p_{5}+p_{6}+p_{7}=2 \mathrm{~K}_{1}+1 \quad u \quad p_{1}+p_{2}+p_{3}=2 \mathrm{~K}_{2}+1
\end{align*}
$$

which ultimately means that an even number is not equal to the difference of two any odd ones, which is impossible and (09) equality.

## 3. Number of representations by the sum of primes, both even and odd not one.

 According to point 1 :$$
\begin{align*}
& p_{1}+p_{2}=p_{4}+p_{5}+p_{6}-p_{7}  \tag{13}\\
& p_{1}+p_{2}+p_{7}=p_{4}+p_{5}+p_{6} \tag{14}
\end{align*}
$$

Starting from 11, any odd number is representable at least in $2 x$
options $2+2+7=11,3+3+5=11$, etc.
According to the ternary problem, we have a recurrent formula for a prime number:

$$
\begin{equation*}
p_{1}=p_{2}+p_{3}+p_{4} \tag{15}
\end{equation*}
$$

Similar to the algebraic sum of 4 primes for 3 primes:

$$
\begin{equation*}
p_{1}=p_{5}+p_{6}-p_{7} \tag{16}
\end{equation*}
$$

What follows:

$$
\begin{equation*}
p_{1}+p_{7}=p_{5}+p_{6} \tag{17}
\end{equation*}
$$

Starting from 14, any even number can be represented in more than one variant:
$14=7+7$,
$14=11+3 \mathrm{etc}$.

## 4. Sum of 2 or more prime numbers.

If the number of primes is more than three, then for an even number, it can be replaced by the sum of two primes and, for odd, by the sum of three.

The sum of 2 or more primes is equivalent for even $n$ -
their number is corresponding to any even number, starting from $2 n$ for even number of terms, and odd, starting $2 n+1$ for odd.

And further:
Difference between the sum of n prime numbers and an odd prime number when $n$ is even, any odd number and vice versa.

Confirmation of this:
$n$-even

$$
\begin{equation*}
p_{1}+p_{2}-p_{3}=p_{4}+p_{5}+p_{6} \tag{18}
\end{equation*}
$$

see point 1 .
n-odd

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}-p_{4}=p_{5}+p_{6}, \quad p_{1}+p_{2}+p_{3}=p_{4}+p_{5}+p_{6} \tag{19}
\end{equation*}
$$

see point 3.

## 5. Gemini is endless.

We have:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}=2 \mathrm{~N} \tag{20}
\end{equation*}
$$

If even: $\quad 2 \mathrm{~N}=2\left(p_{2}+p_{4}\right)+4$, as a result:

$$
\begin{equation*}
p_{1}-p_{2}+p_{3}-p_{4}=4 \tag{21}
\end{equation*}
$$

We substitute instead $p_{1}=5, p_{2}=3, p_{3}=7, p_{4}=5$ first set of twins.
Further $p_{1}=7, p_{2}=5, p_{3}=13, p_{4}=11$ second pair, etc. Thus, if there is a finite pair of twins, then in this case we have a contradiction with the proven sum of two and four prime numbers. That's why twins are infinite!

## Resource

## 1. http://molodyvcheny.in.ua/files/conf/other/33feb2019/67.pdf

## 2. http://www.ijma.info/index.php/ijma/article/view/5973

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13.Vol. 8 No. 01 (2024): Asian Journal of Mathematical Sciences (AJMS) https://www-ajms-in.translate.goog/index.php/ajms/article/view/535? $x$ tr_sl=en\& x tr $t l=r u \& x$ tr $h l=d e \& x$ tr $p t o=w a p p$
